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All the combinations involving 1, 2, 3, 4 only have been used, and thereby 1 has played *with* each 2, 3, 4 once and twice against each. Since the players are identically involved, *mutatis mutandis*, all the other players will have a similar experience.

361. Proposed by C. E. GITHENS, Ph. D., Wheeling, W. Va.

Find three integral values for  $[-10+9\sqrt{-3}]^{\frac{1}{3}} + [-10-9\sqrt{-3}]^{\frac{1}{3}}$ . A solution not involving a cubic is desired.

I. Solution by the PROPOSER.

I. Put  $(-10+9\sqrt{-3})^{\frac{1}{3}} + (-10-9\sqrt{-3})^{\frac{1}{3}} = \frac{1}{2} [(-80+72\sqrt{-3})^{\frac{1}{3}} + (-80-72\sqrt{-3})^{\frac{1}{3}}]$ .

Let  $(-80+72\sqrt{-3})^{\frac{1}{3}} = \sqrt{x} + \sqrt{y}$  and  $(-80-72\sqrt{-3})^{\frac{1}{3}} = \sqrt{x} - \sqrt{y}$ .

Then,  $[6400 - (-15552)]^{\frac{1}{3}} = x - y = 28$ , and, therefore,  $(-80+72\sqrt{-3})^{\frac{1}{3}} = \sqrt{(x+28)} + \sqrt{x}$ . Hence, by raising both sides to the third power,  $(4x+28)\sqrt{(x+28)} + (4x+84)\sqrt{x} = -80+72\sqrt{-3}$ .

Put  $4x+28\sqrt{(x+28)} = -80$ , and  $4x+84\sqrt{x} = 72\sqrt{-3}$ .

Let  $4x+28 = -80$  or factor of  $-80$ ;  $-40, -20, -16, -10, -9, -8, -5, -4, -2$ , and

$4x+84 = 72\sqrt{-3}$  or factor of  $72; 36, 24, 18, 12, 9, 8, 6, 4, 3, 2$ .

Subtracting,  $-56 = -20 - (36)$ .

$\therefore 4x+28 = -20$  and  $x = -12$ .

$$[1] \quad \frac{1}{2} [(-80+72\sqrt{-3})^{\frac{1}{3}} + (-80-72\sqrt{-3})^{\frac{1}{3}}] \\ = \frac{1}{2} [(\sqrt{(x+28)} + \sqrt{x}) + (\sqrt{(x+28)} - \sqrt{x})] = 4, \text{ answer.}$$

II. Similarly with  $(-80+72\sqrt{-3})^{\frac{1}{3}} = \sqrt{(x_0+28)} - \sqrt{x_0}$ , in which the factors  $-80$  and  $24$  added to eliminate the  $4x$ 's produces

$$x_0 = -27 \text{ and } \frac{1}{2} [(\sqrt{(x_0+28)} - \sqrt{x_0}) + \sqrt{(x_0+28)} + \sqrt{x_0}] = -1, \text{ answer.}$$

III. With  $-\sqrt{(x_1+28)} + \sqrt{x_1}$  and  $-16$  and  $72$  as factors as above,  $x = -3$  and  $\frac{1}{2} [-\sqrt{(x_1+28)} + \sqrt{x_1} + (-\sqrt{(x_1+28)} - \sqrt{x_1})]^{\frac{1}{3}} = -5$ , answer.

IV. The root  $-\sqrt{(x_2+28)} - x_2$  produces no two factors of  $-80$  and  $72$  whose difference or sum equals  $\pm 56$ ; hence it is not a root, which is as it should be, for the numerical equation is an example of the "irreducible case" in the Cardan solution of a cubic whose equation is  $x^3 - 21x + 20 = 0$ .

## II. Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Putting  $(-10+9\sqrt{3})\sqrt{-1}=\rho(\cos\phi+\sin\phi\sqrt{-1})$ , we get  $\tan\phi=-\frac{9}{10}\sqrt{3}$ .  $\rho=\sqrt{343}$ , and  $(-10+9\sqrt{3})\sqrt{-1}^{\frac{1}{2}}+(-10-9\sqrt{3})\sqrt{-1}^{\frac{1}{2}}=2\sqrt{7}\cos\frac{1}{3}\phi$ .

$$\therefore 4\cos^3\frac{1}{3}\phi-3\cos\phi+\frac{1}{4}\frac{9}{10}\sqrt{7}=0.$$

By trial,  $\cos\frac{1}{3}\phi=\frac{3}{7}\sqrt{7}$ , and dividing the last trinomial by  $\cos\frac{1}{3}\phi-\frac{3}{7}\sqrt{7}$ , we get  $4\cos^2\frac{1}{3}\phi+\frac{8}{7}\sqrt{7}\cdot\cos\frac{1}{3}\phi-\frac{5}{7}=0$ ; whence  $\cos\frac{1}{3}\phi=-\frac{5}{14}\sqrt{7}$ ,  $\cos\frac{1}{3}\phi=\frac{1}{14}\sqrt{7}$ .

$\therefore$  The three values required are 4, -5, 1.

Also solved by A. M. Harding.

## GEOMETRY.

## 386. Proposed by DANIEL KRETH, Oxford, Iowa.

Construct the triangle, having given, the vertical angle, the sum of the three sides, and the perpendicular.

## I. Solution by H. PRIME, Boston, Massachusetts.

Let  $ABC$  be the required triangle,  $C$  the given angle. On  $AB$  produced take  $BE=BC$ . On  $BA$  produced take  $AF=AC$ . Let  $O$  be the center of circle  $ECF$ . Then we have the angles  $FOE=2(BEC+AFC)=ABC+BAC$ =supplement of  $C$ .

Hence, to construct the triangle, on  $EF$ =the given sum of the three sides form the isosceles triangle  $EOF$ , making  $EOF$ =the supplement of the given vertex angle (or  $OEF=OFE$ =one half the given angle). About  $O$  as center describe the arc  $EF$ . Parallel to  $EF$  and at a distance from it equal to the given altitude draw a line meeting the arc at  $C$  and  $C'$ . Draw  $CA$  and  $CB$ , making the angles  $ACF=AEC$  and  $BCE=BEC$ .  $ABC$  is the required triangle.

## II. Solution by C. N. SCHMALL, New York City, and A. M. HARDING, University of Arkansas.

Construct an angle  $A$  equal to the given vertical angle. Lay off  $AD$  and  $AE$  each equal to *half* the given sum of the sides.

Describe a circle touching these lines in  $D$  and  $E$ . With  $A$  as center and radius equal to the given perpendicular, describe a circle  $MN$ . By a well known method draw a line tangent to *both* these circles touching in  $R$  and  $S$ , respectively, and cutting the sides in  $B$  and  $C$ . Then  $ABC$  is the triangle required.

Proof.  $BR=BD$ ,  $CR=CE$ .

$\therefore BC=BD+CE$ ; hence the triangle has the given perimeter. Also,  $AS$  is perpendicular to  $BC$ ; therefore the triangle has the required altitude. Q. E. D.

Also solved by J. Scheffer and A. H. Holmes.

